Teacher Created Materials
PUBLISHING

Research-Based Curriculum

Teacher Created Materials
Exploring Math

Supplemental Math Program for Intervention and Reinforcement

EXPLORING MATH

An Intervention & Reinforcement Resource

Timely Teaching: Current Approaches to Mathematics Instruction

Introduction

Today's world is changing rapidly, with math proficiency becoming more and more urgent. Current students will find that they need mathematical capability not only for routine tasks of daily living, but to analyze options in insurance and health care, make sophisticated decisions regarding technology, and for a broadening range of careers in engineering and the sciences. Mathematical reasoning and problem solving will be crucial in strengthening our nation's economy, and in competing in the global market.

Fortunately, there is encouraging news regarding gains in mathematical ability. The National Assessment of Educational Progress Mathematics Study 2000, authorized by Congress and administered by the National Center for Education Statistics in the U.S. Department of Education, consists of a report on the educational progress of students in grades 4, 8, and 12. The study sampled approximately 14,000 fourth graders from 742 schools, 16,000 eighth graders from 744 schools, and 13,000 twelfth graders from 558 schools. The major findings are as follows:

- Fourth, eighth, and twelfth grade students had higher than average scores in 2000 than in 1990, the first year in which the current mathematics framework was used.
- In 2000, the percentage of students performing at or above "Proficient"—identified by the NAGB as the level that all students should reach—was 26 percent at grade 4, 27 percent at grade 8, and 17 percent at grade 12. At each grade the percentage of students performing at or above this level was higher in 2000 than in 1990. The exception is twelfth-graders; the percentage of these students reaching the "Basic" level declined.
- Gains were made in student scores across the board, as higher, middle, and lower performing students improved since 1990 at each grade.

What the statistical information suggests is that the Standards developed by the National Council of the Teachers of Mathematics (NCTM) in 1991, and widely adapted as the framework for state standards, are doing the job of improving mathematical instruction. Teacher Created Materials is committed to incorporating the newest Standards from Principles and Standards for School Mathematics (2000) into its newest mathematical curriculum program, ***Exploring Math***, which is designed to supplement and reinforce learning in core mathematics programs.

Teacher Created Materials also recognizes and supports the No Child Left Behind Act (NCLB) signed into law January 8, 2002, which demands accountability and proven classroom methodology based on research. In addition, as stated by NTCM, "It seems reasonable that anyone developing

products for use in mathematics classrooms should document how the materials are related to current conceptions of what content is important to teach and should present evidence about their effectiveness." The purpose of this monograph is to demonstrate Teacher Created Materials' dedication to such accountability by conducting an overview of recent research on teaching mathematics in today's classroom.

Broad Goals, Principles, and Standards

NCTM Goals

In its most recent edition of the Standards, NCTM envisions broad new goals for students that underlie its foundation. These broad goals address the value and purpose of mathematics. If mathematics is to be considered as more than simply a required subject or taught as a mechanical discipline, these broad goals must be incorporated into planning and creation of any mathematical program. The three broad goals are stated as follows:

1. Students shall learn to value mathematics for its cultural, historical, and scientific contribution to the evolution of human existence.
2. Students shall become confident in their ability to use math, and to be aware of their increasing mathematical power to make sense of their world.
3. Students shall learn to reason and communicate mathematically—to know how and when to use the language of mathematics to express ideas and solve problems.

Exploring Math encompasses these goals by including not only appropriate activities, but also instructions for the classroom teacher to guide him or her in the support of these goals. For example, cultural and historically based lessons include a lesson in the grade 2 kit on the Middle Eastern origin of zero, while the grade 4 kit has a lesson on geometry that makes use of Japanese origami. Classroom discussions at all levels include a summary that makes note of the students' increasing knowledge as mathematicians. Finally, there are many opportunities for discussions that encourage reasoning that occur on the one-on-one, dyadic, small group, and whole group levels.

According to NCTM, standards are used in many settings because they ensure the quality of a product. Standards express expectations about goals to be met, and can be employed to rally a group toward new goals. Teacher Created Materials has a history of supporting the use of standards in the development of its supplementary curriculum materials that continues with ***Exploring Math***.

NCTM Principles

Teacher Created Materials has thoroughly examined the current NCTM Principles to ensure that ***Exploring Math*** meets or exceeds the needs of instructors. The NCTM Principles include six themes which are supported by ***Exploring Math*** as follows:

- Equity: high expectations and strong support for all students. Teacher Created Materials has strived to create programs with sensitivity for the diverse needs of today's students. Included in

Exploring Math are suggestions for calibrating many activities to various student needs, whether it is a student with a learning disability who may need additional practice, or a student who needs challenge.

- Curriculum: coherent, relevant, and well articulated across the grades. ***Exploring Math*** adheres to the standards for each grade level. Each ***Exploring Math*** kit is designed specifically for 9 grade levels from K to 8, with each level building upon the skills taught in the previous level and preparing students for the next level.
- Teaching: understanding what students know and need to learn and then challenging and supporting them to learn it well. Teacher Created Materials supports a positive learning environment, which is reflected in the ***Exploring Math*** kits.
- Learning: actively building new knowledge from experience and prior knowledge. The scope and sequence chart included in each ***Exploring Math*** kit assists instructors in understanding what has been and what will be taught at each level.
- Assessment: furnish useful information to both teachers and students. Instructors will find pre- and posttests for each unit of ***Exploring Math***, as well as forms to maintain useful records on each student.
- Technology: influences the mathematics taught and enhances students' learning. The ***Exploring Math*** kits will come with CD-ROM PowerPoint lesson presentations to make use of technology that supports students' learning.

NCTM Standards

The spirit of the new Standards, which demonstrates dedication to mathematical construction and reasoning, while allocating an appropriate place for the acquisition of basic algorithms, is indoctrinated into the ***Exploring Math*** series. The contents of each ***Exploring Math*** kit are driven by the NCTM Standards and cover the following areas of mathematical learning:

Number Sense and Operations
Students should be able to:

- Understand numbers, ways of representing numbers, relationships among numbers, and number systems
- Understand meanings of operations and how they relate to one another
- Compute fluently and make reasonable estimates

For example, at grade 3, students will be required to use a number line to show the relationship between the value of numbers. At grade 6, students will use the same number line to reason and discuss ratios.

Algebra
Students should be able to:

- Understand patterns, relationships and functions
- Represent and analyze mathematical situations and structures using algebraic symbols

- Use mathematical models to represent and understand quantitative relationships
- Analyze change in various contexts

These standards are addressed in Exploring Math in several ways. For example, at grade 1 students will compare the patterns of how many students who own pencils also own crayons. At grade 6, students will analyze changes in number values by plotting equations.

Geometry:
Students should be able to:

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- Specify locations and describe spatial relationships using coordinate geometry and other representational systems
- Apply transformations and use symmetry to analyze mathematical situations
- Use visualization, spatial reasoning, and geometric modeling to solve problems

At grade 1, students can describe the properties of a square; at grade 4, students can describe the properties of a cube and by reason relate them to the properties of a square.

Measurement
Students should be able to:

- Understand measurable attributes of objects and the units, systems, and processes of measurement
- Apply appropriate techniques, tools, and formulas to determine measurements

At grade 2, students will engage in an activity in which they will reason the ratio of pints, quarts, and gallons. At grade 7, students will reason what scale of measurement is appropriate for different types of maps.

Data Analysis and Probability
Students should be able to:

- Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them
- Select and use appropriate statistical methods to analyze data
- Develop and evaluate inferences and predictions that are based on data
- Understand and apply basic concepts of probability

At grade K, students will create graphs relating to favorite foods cut from magazines that will be used in various counting activities. At grade 6, students will reason how to create formulas that predict events, such as which of four colors of cube will be drawn from a container.

Problem Solving
Students should be able to:

- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems

- Monitor and reflect on the process of mathematical problem solving

Students at grade 1 will reason how they can decide how many marshmallows are needed for their small group if they are each going to eat two marshmallows. Students at grade 5 will reason how to change a recipe to serve additional diners.

Reasoning and Proof

Students should be able to:

- Recognize reasoning and proof as fundamental aspects of mathematics
- Make and investigate mathematical conjectures
- Develop and evaluate mathematical arguments and proofs
- Select and use various types of reasoning and methods of proof

Students at grade 3 will demonstrate how they "know" that when they add 53 + 41 that the "9" that they get in the tens place means "90" through discussion and demonstration using real-life materials.

Research Literature Review

Constructing a set of curriculum materials for teaching mathematics requires more than simple adherence to the NCTM Standards, however. In planning ***Exploring Math***, Teacher Created Materials conducted a literature review to ascertain current theoretical thinking in the academic arena, and to glean the most recent ideas about what needs to be incorporated into the pedagogy of mathematics.

Constructivism vs. Traditional Methodology: Integrating Approaches

Not long ago, the term "fuzzy math" came into use. "Fuzzy" referred to the concept that it was more important to understand the process of arriving at a conclusion in mathematical problem solving than it was to obtain the actual answer. The term "fuzzy" was an unfortunate one, because it undercut the importance of the idea of developing conceptual reasoning in mathematics and added fuel to the fire in the debate over whether to teach mathematics in a traditional or reform manner.

In traditional methodology of teaching mathematics, students are exposed to rote learning and mastering of algorithms, which are reinforced through practice and drill. Mathematical operations are taught through procedures that lead the student to the correct answer. This can be likened to a behaviorist approach, in which the stimulus leads to the response, with correct responses being rewarded. This approach was very popular in the mid-twentieth century, in which mathematical learning was quite programmed; computerized learning programs developed later, in the 1970s and 1980s, contributed to this programmed approach. In any case, this is the method by which most parents, teachers, and administrators learned mathematics.

In a paper entitled "Setting the Record Straight," NCTM notes that the traditional method leads to "math avoidance" and "math phobia." The theory behind this is that students were

unable to reason mathematically, so that when novel problems or situations were presented that required conceptualization, they became frustrated and gave up. More importantly, traditional methods left students lagging behind in math compared to students in countries that are our economic competitors. On a developmental level, Posamentier (2003) points out that rote learning at a young age may actually impede creative problem solving, as procedural knowledge is learned on a different side of the brain than mathematical reasoning. Like language, mathematics is an abstraction, and it may be that early problem solving behaviors are critical in being able to truly understand numbers and operations. Thus, there is a current push to introduce algebraic reasoning to kindergartners, long before they are able to produce and answer algebraic equations.

In addition, traditional methods detract from the NCTM principle of "equity," because males and females learn math in different ways. Traditional mathematics methodology favors learners who do not mind working in isolation, which is considered to be more an attribute of male learners. Female students prefer to learn cooperatively in small groups. They are display more interest in mathematics when connections are made between what is learned during mathematics and its utility in other areas. These discrepancies between male and female learners can be addressed by having students interact in pairs and small groups, as well as whole class discussion, and decreasing "answer dependency" to increase the intuitive thinking characteristic of female learners (Morris, 1990).

Recent emphasis in mathematics methodology is on "constructivism," a concept that comes from the field of cognitive psychology and the research of Piaget and Vyotsgy. In this model, learning occurs through *scaffolding* (Vyotsgy), or building one concept onto another, and by attaching meaning to the learned concepts (Piaget). In this way, students become more active learners and better retainers of knowledge. Students are encouraged to use the knowledge they already possess and build upon it. They determine what knowledge they have that is relevant to the problem with which they are presented and apply that knowledge. They then give new meaning to the ideas they are developing (Posamentier, 2003).

The current classroom focus, then, is on problem solving, which Posamentier (2003) defines as "any task that causes children to wrestle or struggle with the mathematics that we want children to learn . . . Problem solving places the focus of attention on ideas and sense-making rather than following . . . directions . . . Solving problems develops the belief in students that they are capable of doing mathematics and the mathematics makes sense."

The move toward using problem solving to teach fundamental mathematics is not as new as educators might believe. In 1969, eminent Stanford mathematician Polya delivered a lecture that has been prescribed and posted on "Mathematically Sane," a Web site devoted to current trends in mathematics teaching. In his lecture he stated, "[T]he

general aim of mathematics teaching is to develop in each student as much as possible the good mental habit of tackling any kind of problem. . . . Mathematics teaching could also develop clarity and staying power . . . the main point in mathematics teaching is to develop the tactics of problem solving." In addition, Ball & Bass (1995) assert, teaching mathematical reasoning is not a "frill." In an article entitled "Making Mathematics Reasonable in School," the authors state that "mathematical reasoning is as fundamental to knowing and using mathematics as comprehension of text is to reading." Reasoning requires applying information that is known to a problem in which the answer is unknown and coming to a new understanding. Mathematical reasoning fertilizes the development of knowledge over time, whereas factual knowledge is static; conversely, possession of factual knowledge does not imply that reasoning has occurred.

Regardless of the origins of current methodology, constructivist approaches to mathematics do demonstrate improvement in students' ability to reason. According to Rowen, (1989), there are three approaches to problem-solving for younger children:
1) Teaching *about* problem solving, which focuses on strategies—the "how to" of figuring out the meaning of a problem and planning a way to solve it. This approach also encourages students to reflect on the solution upon which they have arrived.
2) Teaching *for* problem solving, in which students must apply mathematical approaches to real-life problems; and
3) Teaching *via* problem solving, in which students use mathematical problems to learn new ideas as well as to create bridges between new and already-constructed ideas. Teaching via problem solving can be used to begin a course of study on a mathematical concept new to students, or to investigate previously introduced mathematical concepts.
All three of these strategies are incorporated into ***Exploring Math***. Teacher Created Materials supports the critical need for activities that go beyond rote learning. Activities that require reasoning are woven throughout the ***Exploring Math*** program.

Still, factual knowledge has its place in the mathematics classroom. Ball & Bass (1995) speak of "public knowledge," or the mathematical concepts of which students are aware and that they bring to the classroom, e.g., students may bring awareness that items can be added together, but what is not yet known is why or how this is done. Such basic knowledge is acquired from life experiences in the family and the community. Sfard (1998), in an article entitled "Balancing the Unbalanceable," states that this type of knowledge must then be fostered through problem-solving activities, which then must be translated into algorithms. Sfard then advocates for "reflective practice," which is practice with algorithms that allows students to consider what they are learning. This reflective practice serves as more than just reinforcement of a concept learned through problem-solving. Sfard states that "students like to practice because "they become fluent in technique; [practice] gives them the sense of success and security necessary to proceed . . . Mastery of an action leads

to reflection on the meaning of that action and an increase in understanding leads to new, more complex actions."

Thus, supplemental materials such as ***Exploring Math*** gives students an opportunity for additional "reflective practice" when it is warranted through the use of structured lessons and exercises.

Fostering Communication to Increase Students' Understanding of Mathematics

Aside from the straightforward mathematical standards, the communications standard is also critical. The communications standard requires that students organize their thinking about mathematics and communicate their thinking clearly to peers, teachers, and others. According to Rowan, (1989), communication is essential if students are to integrate new mathematical knowledge.

First and foremost, when students discuss ideas and listen to one another, it helps them to clarify their thoughts, to use definitions and apply them, and to cement conceptual thinking. Second, communication helps students establish a shared understanding, as well as a community of learners that supports their investigations. Communication in the classroom about mathematical concepts helps students confirm their ideas, and they learn that their contribution to understanding these concepts is valuable.

In addition, when communication about mathematics is nurtured in the classroom, it creates a comfortable learning environment. Listening to peers also helps them to confirm that their thinking is "on track." Students feel less threatened and anxious when they have an opportunity to communicate their mathematical ideas to peers, and such sharing is pleasurable. Finally, when students communicate with one another, the teacher has an opportunity to evaluate the students' thinking, and to encourage additional thought in order to verify their investigations.

Communication about mathematics can be encouraged in many ways, but some of the most important include the following:

Using physical materials: When students are presented with real-life material, they have an opportunity to describe the material, tell about its characteristics, create a narrative about a problem which they might encounter with the material, and so forth. ***Exploring Math*** will include multiple activities at each grade level that provide such encounters, in the hope that the activities can be used as a springboard for discussions at multiple classroom levels.

Using interesting and relevant topics: Students are apt to be more engaged when mathematical discussions use information about the students themselves, e.g., their favorite foods, activities, pets, and so forth. With older students, teachers can use statistical information or facts derived from social and political sciences to implement a math-based discussion. In addition, ***Exploring Math*** will provide suggestions on ways to stretch mathematics learning throughout the

day, so that students can see that it is not an isolated subject. When students are made aware, for example, that quite a bit of information in their social studies and science texts are derived from mathematical activities such as statistics, the question of relevance is answered.

Questions: Supply open-ended questions, such as "What does *triangular* mean?" These types of questions pique student curiosity. The activities in ***Exploring Math*** will include suggestions for leading such open-ended discussions, enabling the classroom teacher to focus on tending to the students' discussions rather than to the pressures of having to produce thought-provoking questions on the spot. Most classroom teachers are skilled at asking questions that beg investigation, but such questions can require advance preparation. A program such as ***Exploring Math*** can assist the teacher greatly in this regard.

Writing: Motivate students to write about their mathematical thinking and then share and discuss their writing. The "Problem Solver's Journal" found in ***Exploring Math*** is a major component that supports student writing to explain their solutions to daily problems.

Cooperative and collaborative groups: Use small groups for initial discussion, and then whole class discussions to examine the findings of the small groups. This type of activity is at the heart of many programs created by Teacher Created Materials. ***Exploring Mathe*** will be another such program, with specific plans for teachers to follow regarding one-on-one, dyadic, small group, and whole group discussions.

Listening: Pair students, then ask them to take turns listening to one another explain a mathematical concept and restating what they have heard. Activities such as this will be incorporated throughout ***Exploring Math*** in order to foster discussions. When peers have an opportunity to interact, they can teach one another at their own level. In addition, cooperative and collaborative group discussions and listening activities such as this are the type that is enjoyed by female students, who may experience less math anxiety when they do not have to perform individually for an authority such as the classroom instructor.

Teacher Created Materials has a strong history of including motivational tasks that encourage this type of oral reasoning. ***Exploring Math*** activities direct students to articulate their thinking and reasoning about mathematical operations and concepts. Through this interaction, students recognize and use the language of math, rather than remaining passive receptacles of learning.

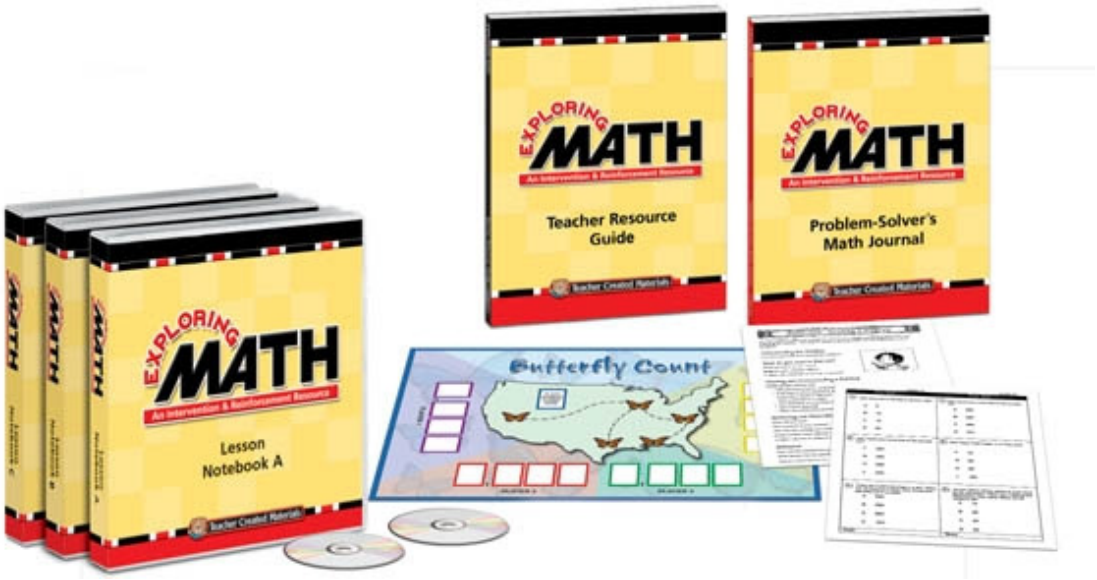
Teacher Created Materials also seeks to help students make the connection between the math that they learn in the classroom and other subjects or daily life activities, as suggested by the North Central Regional Educational Laboratory (NCREL). NCREL suggests that students must be able to go beyond the ability to perform mathematical functions to applying their skills to daily problem solving. This also includes using technology to "enhance, extend, and connect classroom experiences within and outside the classroom." The

activities in ***Exploring Math*** are designed to stretch other areas of the learning day, such as content area instruction. The activities cards all include specific information about science, social studies, or jobs people hold. This provides students with the real world connections they need.

Assessment

Assessment is not a static event. Assessment is an ongoing process that determines what knowledge has or has not been acquired. According to Midgett (2001), a Presidential Awardee in Elementary Mathematics quoted in a paper issued by the North Central Regional Educational Laboratory (NCREL), "Student performance is monitored to promote learning, to adjust instruction, and to report progress." ***Exploring Math*** will include pre- and post-tests for each unit to support student assessment. In addition, the program will include forms for record keeping that will allow the teacher to assess student progress during the course of a unit.

In addition, assessment in ***Exploring Math*** will be aligned with the instructional goals reflected in the standards. That is, students' reasoning skills, as well as their computational skills, will be addressed. As stated by Rowan, Thompson, & Briars (1989), "[M]athematical knowledge entails more than the competencies that conventional assessments in mathematics have tended to emphasize." Students will need to demonstrate their understanding of concepts and their ability to apply this understanding. In this way, students will be exposed to the type of assessment that they will encounter more frequently in the future.



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